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PROJECT TITLE: The Effects of Initial Conditions on the 3-D Topology of Temporally Evolving Wakes,
N00014-90-J-1976
(ARI on 3-D bluff body wakes)

GRANT PERIOD: May 1, 1990 to July 30, 1993

DATE OF THIS REPORT: September 27, 1993

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I. SUMMARY

A three year study of the effect of initial conditions on the topology and fine scale structure of temporally evolving wakes has been completed. Early in the project new topological analysis tools were developed and used to analyze available simulations of the plane mixing layer. During the course of these preliminary studies it was found that certain geometrical features of the fine scale motions in the mixing layer were very similar to those seen in simulations of forced isotropic turbulence. As a consequence, we adopted a two-track approach to the work; one track was to gather together simulation data on as wide a variety of shear flows as possible with the objective of identifying universal feature of the fine scale structure. The second track was to carry out a new direct simulation of the plane wake in which systematic variations of the initial conditions were used to identify structural features of the developed flow which depend on initial conditions.

The fundamental questions we tried to address were: What features of the fine scale motions in turbulent flow are universal, what features are not, and, in the case of the plane wake, what information about the near wake survives far downstream? Interpretation of the computed flow fields was carried out using topological methods. In this approach the time evolution of the flow is followed in terms of joint pdf's of the invariants of the velocity gradient tensor. The invariants can be directly related to the topology of the local velocity field enabling useful comparisons to be made between significant features of the pdf's and corresponding features in physical space. Of particular interest is the second invariant of the rate-of-strain tensor which is directly proportional to the dissipation of kinetic energy. An intermediate asymptotic theory for the evolution of the velocity gradient tensor was developed under this project and used to identify, for the first time, an attractor in the space of tensor invariants. Several of the important universal as well as initial-condition-dependent features of the invariant joint pdf's were explained by this theory.

II. LONG TERM GOALS:

1. To gain improved understanding of the fundamental mechanisms of kinetic energy dissipation in turbulent flow and to identify universal features of turbulent fine scale structure.

2. To develop dynamical models for the time evolution of the velocity gradient tensor.

3. To study the linkage between the late stages of flow development and initial conditions.

4. To develop topological methods for revealing significant flow field features contained in large data sets

III. APPROACH, TASKS COMPLETED

1. Method of approach, survey of simulation data

Studies of the topology of fine scale motions in a variety of flows have been carried out during this project. The cases of interest include the following time developing simulations

(i) Weakly compressible mixing layer
(ii) Incompressible mixing layer
(iii) Weakly compressible wake
(iv) Incompressible wake
(v) Weakly compressible homogeneous shear flow
(vi) Incompressible homogeneous shear flow
(vii) Incompressible channel flow

Discussion of these cases may be found in the list of publications attached to this report (References 4, 5, 7, 8, 11, and 12). In the approach the partial derivatives of the velocity field are determined at every point in the flow. These are used to construct the invariants of the velocity gradient tensor (P, Q, R), the rate-of-strain tensor (P_S, Q_S, R_S) and the rate of rotation tensor (0, Q_W, 0). For incompressible flow the first invariant is zero. For the conditions of the compressible simulations, the first invariant is found to be everywhere small relative to the second and third invariants and so in both cases the local topology at a point is mainly determined by the second and third invariants. The resulting ensemble of (Q,R), (Q_S, R_S) and (Q_S, Q_W) pairs is sorted into bins and presented as number density contours. See for example Figures 7 to 18 in Reference 4. In this form the invariant plots are equivalent to unnormalized joint probability density distributions of the two invariants being crossplotted. The invariants can be directly related to the topology of the local velocity field enabling useful comparisons to be made between significant features of the joint pdf's and corresponding flow structures in physical space. Working in the space of invariants in this fashion is an efficient way of revealing significant structural features of the flow. Of particular interest is the second invariant of the rate-of-strain tensor, Q_S , which is directly proportional to the dissipation of kinetic energy.

2. Modeling the evolution of the velocity gradient tensor

The velocity gradient tensor satisfies a nonlinear evolution equation of the form $dA_{ij}/dt + A_{ik}A_{kj} - (1/3)\,(A_{mn}A_{nm})\delta_{ij}=H_{ij}$ where $A_{ij}=\partial u_i/\partial x_j$ and the tensor H_{ij} contains terms involving the action of cross derivatives of the pressure field and viscous diffusion of the velocity gradient. The homogeneous case ($H_{ij} = 0$) forms a ninth order system of ODE's for the components of the velocity gradient tensor and has been studied by a number of investigators using asymptotic analysis and numerical integration beginning in the early 1980's. In the course of our studies we discovered an exact integral of the equations (Reference 1). The asymptotic behavior of the exact solution has a number of properties in common with observations of direct simulation. In addition the exact solution has enabled us to precisely determine the connection between initial conditions and the asymptotic state of the restricted Euler system.

No one previously has been able to make any general statements about the behavior of the velocity gradient tensor when the assumption $H_{ij} = 0$ is removed. However using results from the restricted Euler solution a new intermediate asymptotic theory for the evolution of A_{ij} has been developed in which no restriction is placed on H_{ij} but the velocity gradient tensor is assumed to evolve toward the form $dA_{ij}/dt=f(t)A_{ij}$ where $f(t)$ is a scalar function of time. The restricted Euler solution predicts a specific $f(t)$ however in the context of the intermediate asymptotic model $f(t)$ is unspecified and can be used to fit experimental (numerical) data. Under this assumption a general relationship can be worked out between the invariants Q and R and the discriminant of H_{ij}. This leads to the notion that Q and R must lie in a region of attraction in the space of tensor invariants (See Figures 3 and 4 of Reference 3). Various regions of this attractor represent local flow fields which can be characterized as either sheet-like or tube like in nature.

3. <u>Direct simulation of temporally evolving planar incompressible wakes</u>

Previous experimental studies of plane wakes produced by an assortment of different wake generators, all arranged to have the same drag, demonstrated strong sensitivity of the turbulent structure of the wake to initial conditions. This sensitivity is observed, to varying degrees, in a wide variety of free shear flows and is one of the most difficult impediments to the development of accurate models. In the present study direct numerical simulations of an incompressible plane wake are used to investigate the linkage between the late stages of wake development and properties of the initial flow field. A series of simulations were carried out on the NAS Cray YMP to explore the effects of various combinations and phasings of 2-D and 3-D laminar disturbances on the transition to turbulence in wakes with Reynolds numbers based on initial halfwidth and deficit velocity of between 500 and 4000. The simulations used a pseudospectral Fourier/Jacobi code developed at NASA Ames. All the simulations were initiated with a Gaussian mean velocity field perturbed by small disturbances periodic in the streamwise and spanwise directions. Some of the preliminary results of these studies are described in Reference 10. Scaling laws and the effects of initial conditions on the structure of invariant joint pdf's are presently under study.

4. <u>Development of a direct Fourier method for simulating incompressible flows with one infinite and two periodic directions</u>

Initially the simulation code for the wake was a modification of an existing code developed for the Cray-YMP. However because of the high level of usage of the YMP the turn around time for the computation was extremely slow and it was decided to move the simulation to the iPSC/860 hypercube. An entirely new spectral code has been written using a novel approach in which a Fourier approximation to the flow in the vortical region is matched to an analytic asymptotic solution for the far field. The algorithm makes use of the fact that the vorticity field in planar free shear flows is compact in the cross-stream direction. This allows the vorticity equations governing the flow to be solved in a fully periodic box, with the non-periodic velocity being calculated by matching to tne known asymptotic solution. Because this avoids nonlinear mapping of the flow domain, direct use of Fast Fourier Transforms in all three spatial directions is possible, resulting in a code which is small, fast, and natural to implement on parallel architectures. Further studies of the plane wake using this code along with comparisons with the previous Cray YMP simulations are presently in progress.

IV. SCIENTIFIC / ENGINEERING RESULTS

The work carried out under this grant has revealed a number of important geometrical features of the fine scale structure of incompressible and weakly compressible inhomogeneous shear flows; The second and third invariants of the velocity gradient and rate-of-strain tensors are strongly correlated; Motions corresponding to high rates of dissipation are found to be characterized by a 3-D rate-of-strain topology which is of the type saddle-saddle-unstable-node; Regions corresponding to high rates of dissipation are found to be characterized by comparable magnitudes of enstrophy and strain with the vorticity vector closely aligned with the intermediate positive strain direction. In the incompressible cases, motions characterized by the highest enstrophy were found to be nearly strain-free (ie, in solid body rotation). Whereas in the compressible cases motions characterized by the highest enstrophy were always imbedded in regions of high strain rate. By studying a variety of flows it has been possible to accumulate a large body of information which supports the hypothesis that in free shear layers the features itemized above are universal (References 4, 5, 6, and 7). Recently in Reference 4 it was discovered that the rate-of-strain topology just described persists to substantially larger scales than was previously realized.

The asymptotic behavior of the restricted Euler solution discussed above reproduces many of the topological features seen in the simulations. Examination of the exact solution of this system provides new insight into the manner in which the fine scale motions of turbulence can evolve to a universal state and the degree to which this state can depend on initial conditions. A new intermediate asymptotic model of the evolution of the velocity gradient tensor shows improved agreement with simulation data and identifies, for the first time, a region of attraction in the space of velocity gradient tensor invariants. Very recently in a study of turbulent channel flow (Reference 5) it has been found that in regions of high dissipation the vorticity vector is aligned approximately 10 degrees away from the intermediate strain direction.

Simulations of plane incompressible wakes demonstrate the important role of subharmonic components in the initial field for the subsequent generation of three-dimensionality.

V. ACCOMPLISHMENTS

1. A three-dimensional topological analysis method based on the joint pdf of local tensor invariants has been developed and found to be a useful way to eluc· late significant features of large flow field data sets.

2. The topological analysis method has been used to identify important geometrical features of the fine scale structure of turbulence in a variety of incompressible and weakly compressible inhomogeneous shear flows. Evidence to date supports the hypothesis that these features are universal.

3. An exact closed form solution has been discovered for a restricted Euler equation for the velocity gradient tensor giving new theoretical insight into the evolution of turbulent fine scale structure to a universal state. An intermediate asymptotic theory suggests for the first time that the velocity gradient tensor evolves to an attractor in the space of tensor invariants.

4. Simulations of plane incompressible wakes demonstrate the importance of subharmonic components in the initial field for the subsequent generation of three-dimensionality. A new spectral code has been developed in which a Fourier approximation to the flow in the vortical region is matched to an analytic asymptotic solution for the far field. This allows direct use of FFT's in all three spatial directions, resulting in a code which is especially well suited to parallel architectures.

V. PUBLICATIONS UNDER ONR GRANT N00014-90-J-1976

P.I. Brian J. Cantwell

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(1) 92-P CANTWELL, B.J. 1992 Exact solution of a restricted Euler Equation for the velocity gradient tensor. *Physics of Fluids A* **4** (4), April pp. 782 - 793.

(2) 92-P SUBBARAO, E.R. and CANTWELL, B.J. 1992 Investigation of a co-flowing buoyant jet: experiments on the effect of Reynolds number and Richardson number. *Journal of Fluid Mechanics* **245**, December pp. 69-90. (Partially supported by this grant).

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(5) 93-PS BLACKBURN, H., MANSOUR, N., and CANTWELL, B. 1993 Topology of fine scale motions in turbulent channel flow. Submitted to *J. Fluid Mech.*

(6) 93-PI SONDERGAARD, J., CHEN, R., SORIA, J., and CANTWELL, B. 1993 Local topology of small scale motions in turbulent shear flows. Manuscript in preparation.

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(7) 91-C SONDERGAARD, J., CHEN, R., SORIA, J., and CANTWELL, B. 1991 Local topology of small scale motions in turbulent shear flows. Proceedings of the Eighth Symposium on Turbulent Shear Flows, Munich.

(8) 92-C SORIA, J., and CANTWELL, B 1992 The identification and classification of topological structures in free shear flows. Proceedings of the IUTAM Symposium on Eddy Structure Identification, Poitiers, France, 12-14 October 1992

(9) 92-C CANTWELL. B. 1992 Geometry of turbulent fine scale structure. Proceedings of the 11th Australasian Fluid Mechanics Conference, Hobart Aus 14-18 December 1992.

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(13) 91-C SONDERGAARD, R., CHEN, J., and CANTWELL, B. 1991 Local vorticity-strain alignment,rate-of-strain and pressure-strain distributions in direct numerical simulations of compressible and incompressible flow. Proceedings of the 44rd Annual Meeting of the Division of Fluid Dynamics of the American Physical Society, Arizona State University.

(14) 92-C BLACKBURN, H., MANSOUR, N., and CANTWELL, B. 1992 A study of fine scale motions in turbulent channel flow. Proceedings of the 45rd Annual Meeting of the Division of Fluid Dynamics of the American Physical Society, Florida State and A & M University.

(15) 93-C SONDERGAARD, R., MANSOUR, N., and CANTWELL, B. 1993 A direct Fourier method for the simulation of incompressible flows with one infinite and two periodic directions using asymptotic matching of velocities. Proceedings of the 46th Annual Meeting of the Division of Fluid Dynamics of the American Physical Society, Albuquerque, New Mexico.